Extended nonlocal games and monogamy-of-entanglement games

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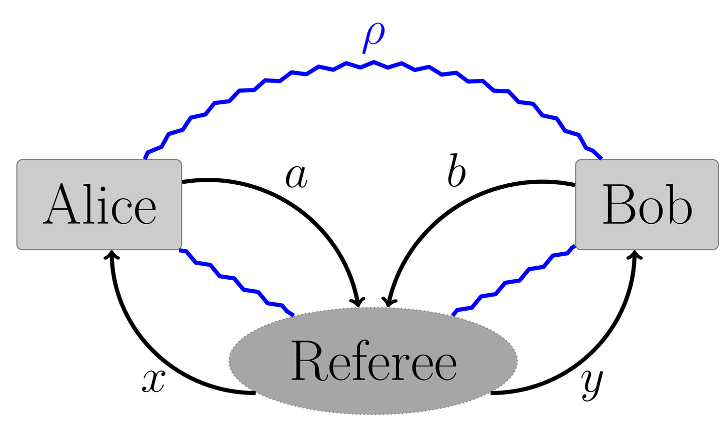
Extended nonlocal games

An **extended nonlocal game** is a generalization of nonlocal games where the *referee also holds a quantum system*, provided to it by Alice and Bob at the start of the game.



Preparation stage: Alice and Bob supply the referee with a quantum system.

Question stage: Referee randomly selects questions $x \in X$ for Alice, $y \in Y$ for Bob according to distribution π.

Answer stage: Alice responds with $a \in A$, Bob responds with $b \in B$.

Evaluation stage: Pay-off is determined by an **observable** $V(a, b|x, y) \in \mathrm{Herm}(\mathbb{C}^m)$ where m is the dimension of the referee's quantum system.

Monogamy-of-entanglement games

A **monogamy-of-entanglement game** [3] is a type of extended nonlocal game where the referee performs a measurement and accepts iff Alice's output, Bob's output, and the Referee's measurement output are the *same*.

The *BB84 monogamy-of-entanglement game*, G_{BB84}, is defined in the following way. Let $m = 2$, let $X = A = \{0, 1\}$, and define

$$\text{For } x = 0: \quad R(0|0) = |0\rangle\langle 0|, \qquad R(1|0) = |1\rangle\langle 1|,$$
$$\text{For } x = 1: \quad R(0|1) = |+\rangle\langle +|, \qquad R(1|1) = |-\rangle\langle -|.$$

Then $G_{\mathsf{BB84}} = (\pi, R)$ where $\pi(0) = \pi(1) = 1/2$.

- $\omega(G) = \omega^*(G) = \cos^2(\pi/8)$ [3].
  - Entanglement does not help Alice and Bob in G_{BB84}.
- $\omega(G^n) = \omega^*(G^n) = (\cos^2(\pi/8))^n$ [3].
  - Strong parallel repetition holds for G_{BB84}.

Standard quantum strategies

A **standard quantum strategy** consists of finite-dimensional complex Hilbert spaces $\mathcal{A}$ and $\mathcal{B}$ as well as the following:

Shared state: $\rho \in \mathrm{D}(\mathcal{R} \otimes \mathcal{A} \otimes \mathcal{B})$

Measurements: $\{A_a^x\} \subset \mathrm{Pos}(\mathcal{A})$, $\{B_b^y\} \subset \mathrm{Pos}(\mathcal{B})$.

The expected **pay-off** for a quantum strategy is:

$$\sum_{(x,y)\in X\times Y} \pi(x, y) \sum_{(a,b)\in A\times B} \left\langle V(a, b|x, y) \otimes A_a^x \otimes B_b^y, \rho \right\rangle.$$

The **quantum value** ($\omega^*(G)$), is the supremum of the pay-off over all quantum strategies for G.

Unentangled strategies

In an **unentangled strategy** Alice and Bob provide the referee with a pure state $\rho \in \mathrm{D}(\mathcal{R})$ and Alice responds to $x \in X$ with $a = f(x)$ and Bob responds to $y \in Y$ with $b = g(y)$.

From convexity the **unentangled value** is achieved by some deterministic strategy, and may be represented as

$$\omega(G) = \max_{f,g} \lambda_{\max} \left(\sum_{(x,y)\in X\times Y} \pi(x, y) V(f(x), g(y)|x, y) \right)$$

where $\lambda_{\max}$ is the largest eigenvalue and where the maximum is over all functions $f : X \to A$ and $g : Y \to B$.

Commuting measurement strategies

A **commuting measurement strategy** consists of a finite-dimensional complex Hilbert space $\mathcal{H}$ as well as the following:

Shared state: $\rho \in \mathrm{D}(\mathcal{R} \otimes \mathcal{H})$

Measurements: $\{A_a^x\} \subset \mathrm{Pos}(\mathcal{H})$, $\{B_b^y\} \subset \mathrm{Pos}(\mathcal{H})$

satisfying the constraints that $[A_a^x, B_b^y] = 0$, for all $x \in X$, $y \in Y$, $a \in A$, and $b \in B$.

The expected **pay-off** for a commuting measurement strategy is given by

$$\sum_{(x,y)\in X\times Y} \pi(x, y) \sum_{(a,b)\in A\times B} \left\langle V(a, b|x, y) \otimes A_a^x B_b^y, \rho \right\rangle.$$

The **commuting measurement value** ($\omega_c(G)$), is the supremum of the pay-off over all commuting measurement strategies.

Extended NPA hierarchy

- Not known how to calculate $\omega^*(G)$ in general.

NPA hierarchy: The NPA hierarchy is a heuristic to upper bound $\omega^*(G)$ for nonlocal games [1, 2].

Extended NPA hierarchy: We introduce the extended NPA hierarchy that places upper bounds on $\omega^*(G)$ for extended nonlocal games.

- $\omega^*(G) \overset{?}{=} \omega_c(G)$ is a big open question, and the NPA hierarchy yields increasingly better approximations on $\omega_c(G)$.

Entangled vs. unentangled strategies for monogamy games

Result: For any monogamy-of-entanglement game G where $|X| = 2$ it holds that $\omega(G) = \omega^*(G)$.

- Class of monogamy-of-entanglement games where entanglement *does not* help the players.

Result: There exists a monogamy-of-entanglement game G where $|X| = 4$ where $\omega(G) < \omega^*(G)$.

- Example of monogamy-of-entanglement game where entanglement *does* help the players

Parallel repetition of monogamy games

Result: For any monogamy-of-entanglement game defined in terms of projective measurements where $|X| = 2$, it holds that $\omega^*(G^n) = \omega^*(G)^n$.

References

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